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QUARK CONDENSATE IN THE INTERACTING PION-NUCLEON MEDIUM AT FINITE TEMPERATURE AND BARYON NUMBER DENSITY

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The chiral condensate in the interacting pion-nucleon system, $\langle \bar{q}q \rangle^*$, is investigated at finite temperature and density. The analysis has been done within the conventional hadron dynamics on the base of the Weinberg Lagrangian. At zero temperature and a finite nucleon density the interaction corrections increase the ratio $\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle_0$ at the level of $\simeq 10\%$. At finite value of the temperature, the thermalized pion population tends to compensate even this small effect, so that at high value of the temperature the in-medium chiral condensate becomes close to that in the free pion-nucleon gas.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Кварковый конденсат во взаимодействующей пион-нуклонной среде при конечной температуре и барионной плотности

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Киральный конденсат $\langle \bar{q}q \rangle^*$ исследуется во взаимодействующей пион-нуклонной среде при конечной температуре и плотности барионного заряда. Анализ проведен на основе лагранжиана Вайнберга. Учет взаимодействия приводит к слабому (порядка 10%) увеличению отношения $\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle_0$ при нулевой температуре. Однако при конечной температуре этот эффект компенсируется вследствие присутствия термализованных пионов и их взаимодействия с нуклонными источниками. При высоких температурах значение кирального конденсата во взаимодействующей среде почти не отличается от его значения в идеальном газе.

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I. Introduction

The investigation of hadron properties in the hot and dense nuclear matter is an important and interesting problem of the modern nuclear physics. Very likely, the effective hadronic parameters, the mass or decay width, could differ significantly from their values in the free space, depending on the temperature and density of the environment.

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Furthermore, the critical phenomena, such as formation of the quark-gluon plasma and the chiral symmetry restoration, are expected to occur in the extremely hot and dense matter.

One of the modern and widely used methods to study the in-medium effects is the QCD Sum Rules (QCD SR) [1], extended to the case of finite temperature and density [2]. While being successful in describing the hadron properties in free space, the QCD SR approach is also believed to be a useful tool for the theoretical investigation of hadron spectra in hot and dense nuclear matter. The reliable way is to describe the medium-nonaffected short-range dynamics in terms of quark and gluon degrees of freedom, while the long-range and medium-sensitive dynamics is described by introducing the temperature- and density-dependent in-medium condensates $\langle \bar{q}q \rangle^*$, $\langle G^{\mu\nu} G_{\mu\nu} \rangle^*$, etc. It has been made, for example, in Ref.[3], where the authors have evaluated, as input of the Sum Rule, the expectation values of quark and gluon operators taken over the states of the free pion gas at finite temperature. That scheme is reasonable for the investigation of hadron properties at relatively low temperatures and/or densities, whereas for a highly compressed and heated matter, the free gas approximation is certainly not relevant. To extend the consideration to higher values of the temperature and density, it is necessary to evaluate the thermal averages of the QCD operators in a strongly interacting system.

This problem is not new, in fact. The chiral condensate $\langle \bar{q}q \rangle^*$ in the hadronic medium at finite baryon number density was investigated in Refs.[4]. Another case, the interacting pion gas at finite temperature, was considered in Ref.[5]. In the present work, we study one more possibility, the interacting pion-nucleon gas at finite temperature and baryon number density, and evaluate the thermal average of the operator $\bar{q}q = \bar{u}u + \bar{d}d$, using the conventional hadron dynamics.

II. The Model

According to the Feynman–Hellmann theorem, the in-medium quark condensate is related to the free energy density (thermodynamical potential) $\Omega(T, \mu)$

$$\langle \bar{q}q \rangle^* = \frac{\partial}{\partial m_q} \Omega(T, \mu),$$

where m_q is the current quark mass. This form may be rewritten so as to separate the vacuum and matter parts:

$$\langle \bar{q}q \rangle^* = \frac{\partial}{\partial m_q} \Omega(0, 0) + \frac{\partial}{\partial m_q} (\Omega(T, \mu) - \Omega(0, 0)) \equiv$$

$$\equiv \langle \bar{q}q \rangle_0 + \frac{\partial}{\partial m_q} \Delta\Omega(T, \mu). \qquad (3.1)$$

We shall calculate the quantity $\Delta\Omega(T, \mu) \equiv \Omega(T, \mu) - \Omega(0, 0)$ within the conventional hadron dynamics. Therefore, $\Delta\Omega(T, \mu)$ explicitly depends on the effective hadron parameters (masses and coupling constants) rather than on the quark mass m_q. This means that to calculate the quark-mass derivative, one should employ the «chain rule» [4]:

$$\frac{\partial}{\partial m_q} \to \frac{\partial M_N}{\partial m_q}\frac{\partial}{\partial M_N} + \frac{\partial m_\pi^2}{\partial m_q}\frac{\partial}{\partial m_\pi^2} + \frac{\partial g_\pi}{\partial m_q}\frac{\partial}{\partial g_\pi} + \dots$$

Neglecting the dependence of the nucleon-meson couplings on the current quark mass, one can write:

$$\langle \bar{q}q\rangle^* = \langle \bar{q}q\rangle_0 + \left(\alpha_N \frac{\partial}{\partial M_N} + \alpha_\pi \frac{\partial}{\partial m_\pi^2} \right) \Delta\Omega^*(T, \mu) =$$

$$= \langle \bar{q}q\rangle_0 + \alpha_N \Delta \langle \bar{N}N\rangle^* + \alpha_\pi \frac{1}{2} \Delta \langle \pi_a^2 \rangle^*, \qquad (3.2)$$

where the symbol $\Delta \langle Q \rangle^*$ stands for a matter contribution to the statistical average $\langle Q \rangle^*$ (so that $\Delta \langle Q \rangle^* \to 0$ as $T \to 0$, $\mu \to 0$), and $N(\bar{N})$ and π_a are the nucleon and pion field operators, respectively. The coefficients $\alpha_{N,\pi}$ are defined by

$$\alpha_N = \frac{\partial M}{\partial m_q} = \langle N | \bar{q}q | N \rangle, \qquad \alpha_\pi = \frac{\partial m_\pi^2}{\partial m_q} = \langle \pi | \bar{q}q | \pi \rangle,$$

and can be found by using the Gell-Mann–Oakes–Renner (GMOR) relation [6] and the definition of the nucleon Σ_N-term [2,3,7]:

$$\alpha_N = -\frac{\Sigma_N}{f_\pi^2 m_\pi^2} \langle \bar{q}q\rangle_0, \qquad \alpha_\pi = -\frac{1}{f_\pi^2} \langle \bar{q}q\rangle_0.$$

The final expression of the ratio $R_{\bar{q}q} \equiv \langle \bar{q}q\rangle^* / \langle \bar{q}q\rangle_0$ reads:

$$R_{\bar{q}q} = 1 - \frac{\Sigma_N}{f_\pi^2 m_\pi^2} \Delta \langle \bar{N}N\rangle^* - \frac{1}{2f_\pi^2} \Delta \langle \pi_a^2 \rangle^*. \qquad (3.3)$$

To calculate the «matter» parts of the averages $\langle \bar{N}N\rangle^*$ and $\langle \pi_a^2 \rangle^*$, we employ the Weinberg Lagrangian [8] that successfully combines the chiral symmetry predictions with low-energy $\pi - N$ phenomenology [9]. In this paper, we restrict ourselves to the second order in interaction and keep terms up to the third order in hadronic densities and their derivatives. The relevant piece of the Weinberg Lagrangian is of the type:

$$\mathcal{L}_{\text{int}} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3,$$

$$\mathcal{L}_1 = \frac{g_\pi}{2M_N} \bar{N} \gamma^5 \gamma^\mu \tau N \partial_\mu \pi, \qquad \mathcal{L}_2 = \frac{i}{4f_\pi^2} \bar{N} \gamma^\mu \tau (\pi \times \partial_\mu \pi) N,$$

$$\mathcal{L}_3 = \frac{-g_N}{8M_N f_\pi^2} \bar{N} \gamma^5 \gamma^\mu \tau N \pi^2 \cdot \partial_\mu \pi. \qquad (3.4)$$

While performing the calculations, we neglect antiparticle states in the nucleon propagator. In this connection, we use the nonrelativistic reduction of the interaction vertices, given in the textbook [10].

III. Numerical Results and Discussion

For the numerical computation, we use the value of the sigma-term $\Sigma_N = 46$ MeV and $g_\pi^2/(4\pi) = 14.3$. Note that the $\Delta\,\langle\bar{N}N\rangle^*$ average in (3.3) is the scalar nucleon density, which in the nonrelativistic limit is reduced to the nucleon number density. Since we treat the nucleon density ρ as an independent variable, we replace the $\Delta\,\langle\bar{N}N\rangle^*$ by ρ and concentrate on the evaluation of the $\Delta\,\langle\pi_a^2\rangle^*$.

Let us first consider the zero temperature limit, when the thermalized pions disappear and the interaction exhibits itself in the nucleon-nucleon correlations. The diagram giving contribution at $T = 0$ is shown in Fig.1 (a), and represents the $\bar{q}q$ content of the exchanged pions. The numerical result at $T = 0$ is displayed in Fig.2 (upper curves), where the dashed line represents the ideal gas contribution and the solid line is the result of the complete calculation. Our result is in agreement with that of Ref. [11], where it was obtained from the calculation of the exchange Fock energy in the static approximation. Exchanged pions effectively grow the total in-medium value of the chiral condensate, but this correction is rather small and at the saturation density ρ_0 it does not exceed 10% of the ideal gas contribution.

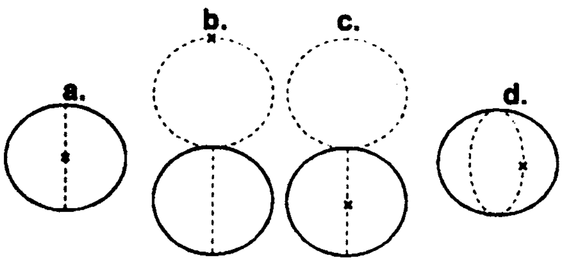

Fig.1. The interaction corrections contributing to the ratio $\langle\bar{q}q\rangle^*/\langle\bar{q}q\rangle_{vac}$. Solid and dashed lines denote nucleons and pions, respectively, and the cross denotes the operator insertion

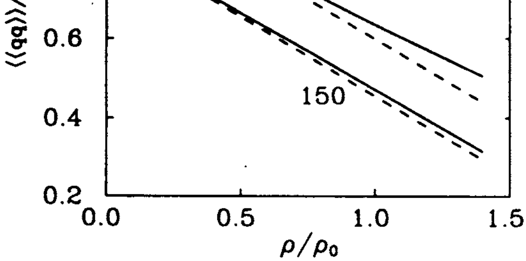

Fig.2. Density dependence of the ratio $\langle\bar{q}q\rangle^*/\langle\bar{q}q\rangle_{vac}$ at the different values of the temperature. The dotted line is the ideal gas contribution and the solid line is the result of the full calculations

The result at $T = 150$ MeV is represented by the lower curves in Fig.2. It is seen from the figure that at a high value of temperature the interaction effect becomes even more weak. Recall that the contribution originated from the thermalized pion population is regulated mostly by the temperature. The positive contribution from diagram (a) is partially compensated by the contribution of diagrams (b, c), which is expected from the comparison of $\mathcal{L}_1$ and $\mathcal{L}_3$ (3.4). Other terms originated from the $NN\pi\pi$ interaction ($\mathcal{L}_2$ in (3.4) and diagram (d) in Fig.1) give a positive contribution, thus effectively increasing the total in-medium content of $\langle \bar{q}q \rangle^*$. Despite these terms, the resulting behaviour of the chiral condensate in the interacting pion-nucleon medium at finite temperature and density is decreasing in shape.

We would like to mention the important role of other hadronic degrees of freedom which are not included into the present consideration. Inclusion of other types of $N - N$ interaction (e.g., the repulsive vector-meson exchange) might alter the final results, but this question is beyond the scope of the present paper.

IV. Summary

In summary, we have estimated the chiral condensate $\langle \bar{q}q \rangle^*$ in the interacting pion-nucleon matter at finite temperature and baryon number density. The calculations are performed on the base of the conventional hadron dynamics up to the second order in interaction and the third order of hadronic densities and their derivatives. At zero temperature and of a finite nucleon density the interaction corrections slightly increase the ratio $\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle_0$. However, at finite temperature this effect is compensated by the contribution originated from the thermalized pions and their interaction with nucleon sources. This compensation is a consequence of the dynamics, based on the chiral symmetry.

Note that our results might be altered in a more realistic description. Namely, other light mesons, as well as delta isobars in the baryonic sector, should be included into consideration within conventional hadron dynamics based on the chiral symmetry. Nevertheless, relying on the present schematic model we are able to figure out hints for interesting phenomena originatd from the finite temperature effects.

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